Stochastic vs. Deterministic Water Market Design: Some Experimental Results

Stephen R Starkey, Shane Dye, E Grant Read, Rosemary A Read
Department of Management
University of Canterbury
Christchurch, New Zealand
stephen.starkey@pg.canterbury.ac.nz, shane.dye@canterbury.ac.nz, grant.read@canterbury.ac.nz, rosie.read@gmail.com

***Abstract*— This paper evaluates a range of market design options and system configurations for a simplified water allocation market. The main focus is to compare market performance of stochastic versus deterministic market configurations. Results and conclusions are presented.**

Keywords- Market Design; Stochastic Markets; Water Markets; CDDP; Reservoir Management.

I. Introduction

Electricity markets are now commonplace and many reservoirs are operated by hydro generators in that context. Many other reservoirs are operated in non-market contexts, some for mixed uses, including for electricity generation, often under traditional sharing rules. Many authors have proposed the development of markets for water, as a commodity in its own right, for whatever use. Market implementation progress has been made in various parts of the world, including Chile (Bauer 2004), California, Spain (Easter et al. 1998) and Australia (Bjornlund et al. 2002), but a standard market design has not yet emerged. Calvatrava and Garrido (2003) provide a good review of prior water market models, noting that the existing literature paid little attention to the change in benefits due to variations in water availability. They develop one such model there and another in Calvatrava and Garrido (2005).

A particular issue is that water is a storable, either explicitly in reservoirs, or implicitly in aquifers, which are not owned by any market participant. Thus, unlike the electricity market, it does not seem possible to simply rely on spot market trading, complemented by financial contracting, under the assumption that reservoirs will be operated by participants outside the market. The market itself must determine reservoir management strategy, allocating water not only between participants in one time period, but between time periods.

Raffensperger et al. (2010) described a deterministic market design for a situation where irrigation water is drawn from a joint aquifer. Read et al. (2012) describe a somewhat analogous market design for gas. In most South American electricity markets, the market/system operator actually determines reservoir releases, but in that case there is no bidding by reservoir owners. Read et al. (2012) discuss a proposal to modify that arrangement by allowing participants to offer their capacity on the basis of a virtual model. But none of these arrangements involve participants bidding to buy and sell water, now and in the future, stored in a common reservoir. The study reported here considers generic designs for such a market, assuming generic demand curves for participants who require water for unspecified purposes. Thus it is applicable to situations in which the demand is for urban water supply, electricity generation, agricultural use, or any combination of these. It also models situations in which all water may be sold into the system, from other systems, or from a desalination plant, for example.

Starkey et al. (2011) proposed a range of water market design options ranging from reliance on a simple spot market, complemented by financial contracting, through to relatively complex models involving stochastic bids, referring to a hydrological index. This paper reports some experimental results from simulation of the some of those market designs. A critical issue is obviously the way in which uncertainty about future inflows, and demands, will evolve. There has been surprisingly little literature reporting on the differences that might be expected in reservoir operation policy between deterministic and stochastic optimisation approaches. Read and Boshier (1989) experimented with a single reservoir model of the New Zealand hydroelectricity system, and showed that the deterministic models were significantly biased towards holding too little water in storage, leading to significant shortages, at high cost. Thus we might expect similar outcomes from a water market based on deterministic optimisation.

II. Market Context

Here, water market designs are compared using a simulation system evaluating market performance in equilibrium. We assume perfect competition between risk neutral participants. Importantly, participant behaviour should be unaffected by contract position under these assumptions. Thus we do not examine, here, the question of whether the forward market solutions produced by our model are used to form contracts, either physical or financial, or are merely seen as indicative by all parties. We are not (yet) trying to determine likely participant behaviour, merely asking what the minimal information requirements might be for the market to operate with reasonable efficiency, assuming all participants know, and truthfully reveal, their present and future preferences. Clearly, participants do not know the future, and we do not model them as knowing the future. We do assume, though, that they know what their demand curve would be in any particular future hydrology state. We do not model any error in this prediction, so as to eliminate a source of noise in our experimental design.

We assume a single reservoir for the storage system, with a market equilibrium reservoir management policy determined using Stochastic Constructive Dual Dynamic Programming (SCDDP), as described by Dye et al. (2012). Participants are assumed to provide bids, not just for the current period, but for all future periods, with those bids potentially being dependent on the hydrology state. SCDDP does not just determine a single market clearing solution, but the whole range of market clearing solutions that would apply under all possible future circumstances. This allows us to determine the equilibrium performance of the system, over the entire distribution of inflows, using either Monte Carlo simulation, or an iterated convolution process. The probability distributions of prices and annual net welfare are then used to compare the performance of different market designs.

While the system can address more general issues, the focus here is on a single research question: Under what set of system characteristics might a market design assuming a deterministic representation of the evolution of future reservoir management be 'sufficient', even though stochasticity is clearly a major factor in the real world?

With the market cleared periodically, the allocation between periods comes down to a crucial question of how much of the available water to store for future release.

III. Market Design Options

Starkey et al. (2011) outline an evolving research framework for considering market design options for water trading in a stochastic environment. These design options are being investigated on an ongoing basis by the Department of Management at the University of Canterbury. The options proposed include the following.

A. *Spot trading only*

Bids are for the current period only and a reservoir operator determines the water to store for later periods. The storage decision will, necessarily, be based on historic bid behaviour only. This option is not considered.

B. *Spot trading with deterministic market simulation*

To improve the storage decision indicative bids are used to value future demands. Bids are based on the expected inflows for each period. For participants, this means providing one bid-stack for each future period. This option is tested using deterministic CDDP to clear the market. Stochastic inflows are assumed for the market performance evaluation.

C. *Spot trading with multi-scenario market simulation*

Here, indicative bids for future periods are based on a given scenario tree or probability distribution of inflows. For participants, this means providing one bid-stack for each period for each scenario. Assuming that water requirements are not dependent on past allocations reduces the future bid requirements to a bid-stack for each modelled inflow (or Markov state) each period This option is tested using SCDDP to clear the market using stochastic independence or a Markov chain to model the inflow distribution.

D. *Spot and multi-scenariofuture trading*

Here, some form of contingent futures trading is used to make future water allocations. That is, bids for future periods are used to form firm contracts, subject to the actual inflows experienced. Under the assumptions of perfect competition and risk neutral participants the market outcomes will be the same as the previous option.

E. *Spot and scaled future trading*

Bids for future releases are adjusted according to some declared mechanism depending on the actual inflows received. This reduces the required bids to one bid-stack per period with an adjustment factor. This option is not tested here.

F. *Design options tested*

The focus here is to compare outcomes from deterministic and stochastic market design options. In terms of the options above, this involves comparing market design option B with options C or D. In addition, we allow two options for the stochastic market, which is assuming stochastic independent inflows or a Markov chain 1-lag structure.

IV. Test System Characteristics

The test system model involves only one reservoir, experiencing varying levels of inflow (F), which may be Deterministic (D), Stochastic Independent (SI), or governed by a Markov Chain (MC). Harvested water may be released (R) to meet demand needs, or stored (S) until a later period. Rather than test a variety of inflow patterns etc. against a reservoir of a fixed capacity, we explore performance across the same range of flow/capacity relativities by keeping the inflow pattern constant (although scaled and shifted as explained below) and varying the reservoir characteristics.

Some care has been taken to set up the system parameters in such a way as to facilitate comparisons between cases in which inflows are assumed to behave differently (D vs. SI, vs. MC), arrive at different times of the year relative to demand, scaled up or down, etc. To minimise noise in our experiments, and simplify the SCDDP algorithm, we also prefer that scaled values be integer. Specifically, then, we assume 12 months of 4 weeks each, with expected weekly flows of 21 units, giving an Expected Monthly Inflow (EMF) of 84 units, and annual flows of 1008 units. This total is thus easily divisible by 2, 3, 4, 6, 7 and 12, to produce reservoir sizes etc. corresponding to quarterly, weekly etc. periods. For simplicity, we assume that lower release/storage bounds are both zero. Our primary focus is on the upper bounds, for storage (SMAX), and (monthly) release (RMAX), capacities and their relativity. For realistic systems, we expect SMAX $\geq$ RMAX $\geq$ EMF. Here we let SMAX range up to the annual expected inflow of 1008, and (monthly) RMAX range up to SMAX. We assume no losses from the reservoir, inflows take only non-negative values, and spill is always possible, but has no economic value, or penalty.

Since inflow varies across time in seasonal patterns, and is naturally stochastic, reservoir storage has two main functions. First it allows water to be stored from seasons when it is expected to be relatively abundant to seasons with relatively short supply. Second, it allows the system to absorb fluctuations due to uncertainty. The first function would be required in a situation with seasonality but no uncertainty, while the latter would be required in a situation with uncertainty, but no seasonality. We wish to test the interaction of storage capacity with these two factors (seasonality and uncertainty), independently.

We model seasonal flows using a sine curve and two seasonality magnitudes were used, 'high' and 'extreme'. For extreme seasonality each month's expected inflow (SMF) ranges from 162, at the height of the wet season, to 6, in the depths of the dry season. Inflow variation for each month is modelled as a seven point, discrete symmetric distribution about the month's SMF. The inflow distribution is scaled in proportion to the month's SMF, when SMF = 162, the month's inflow range is from 69 to 255,but only 3 to 9 for an SMF of 6. This provides a symmetric distribution, with the same EMF across all experiments, without creating negative flow values. High seasonality has an SMF ranging from 147 to 21.

The Markov chain states are the inflows from the previous periods. In the particular experiments reported here, a relatively high weight was applied to the leading diagonal probability values to simulate 'sticky' states. Stochastic independent probabilities are generated from the equilibrium state of a Markov chain. Deterministic cases replace the inflow distributions by the expected inflow (SMF) each month.

In these high level experiments, we do not distinguish individual suppliers or consumers of water, but deal with net aggregate intra-period demand curves for water, downstream from the reservoir. We are only concerned with overall market performance; not individual participant interactions. Thus net demand curves are modelled for various constant elasticities, and we assume that (collectively) participants actually offer these curves into the market, without distortion. For convenience, the curves have all been scaled to give a price of 1, if release volume is set to the month's standard demand quantity (SDQ), regardless of elasticity. The SDQ values follow a sinusoidal pattern with the same range of values as the SMF, that is from 6 to 162, symmetrical about 84. We have modelled demand curves for a wide range of elasticities (-16,-8,-4,-2,-1,-0.5,-0.25,-0.125, and -0.0625). A constant discretization pattern was used for each curve.

Finally, the need for storage is not driven by seasonality, per se, but by the mismatch between seasonal patterns of supply and demand. If seasonal demand variation was perfectly in phase with seasonal inflow variation then, ignoring uncertainty, inflows could be simply passed through the system with no storage required. We model varying requirements for inter-seasonal storage by applying a phase shift, to offset the demand curve data, relative to the inflow data. We consider only one case in this paper, where demand is highest in the dry season, namely six months out of phase with inflow.

V. MODELLING SUITE

The experiments reported here used the market testing suite described in Dye et al. (2012). This system uses SCDDP to determine market equilibrium, assuming perfect competition. Output probability distribution functions (in equilibrium) for storage levels, market clearing prices and other outputs of interest are directly computed from the SCDDP results. The system was implemented in Matlab. Dye et al. (2012) also describe an Excel version.

The market testing suite allows different intra-period decision policies, including heuristic decision rules. We tested two extreme decision policies. The first, termed the "informed" policy, assumes the intra-period flow is observed sufficiently early within a period to allow the release to be based on this. The other extreme is termed the "conservative" policy. This assumes release decisions are based only on the current storage level and that water arriving in a period cannot be released until a subsequent period.

VI. EVALUATING MARKET PERFORMANCE

For each combination of system parameters, the reservoir release policy, representing the set of all possible market clearing solutions for a market design, is determined by the SCDDP algorithm described by Dye et al. (2012). The performance of that market design is then determined by either Monte Carlo simulation, or by applying a convolution algorithm to produce the full probability distribution over all storage (and Markov) states. Either way, the simulation represents performance under the assumed reality, which the market-clearing model may fully account for, or may simplify. Thus, if reality is described by a Markov chain, the market-clearing optimisation model may also assume a Markov chain, it may ignore correlations and assume Stochastic Independent inflows, or it may ignore uncertainty entirely, and assume deterministic inflows.

It would make no sense to assume a Markov chain in the market-clearing optimisation model, though, if there were no correlations in the simulated reality. Of course "reality" will be more complex again.

Key outputs from the simulation include cumulative distribution functions for benefits, and prices. These can be used to compare both the expected value and range of outcomes from different market designs, assuming the same system parameters, and simulated reality. Inflow/demand parameters have been carefully chosen so that, for an unconstrained case, with perfect foresight, the expected volume weighted average prices (VWAP) should be 1. The change in market value can also be calculated. The question is, though what this change should be compared with, given that assuming constant elasticity demand curves always makes the total annual welfare infinite. The "value of the market" (i.e. the volumes sold, valued at market prices) provides a reference point which participants are likely to relate to. For an unconstrained case, with EMF=84, that value will be 1008. So a change in total benefit of 10 would represent 1% of total market value.

VII. COMPUTATIONAL EXPERIMENTS AND RESULTS

A number of preliminary computational experiments were run to compare deterministic and stochastic market design options for various system configurations. The experiments were run over the stylised test system descried above. This set of experiments focuses on three system parameters: the reservoir storage capacity, its release capacity and the demand elasticity.

1) *Experiments*

Two sets of experiments were run. Set A used only high seasonality, and kept storage and release capacities constant and varied the demand elasticity. Set B kept the demand elasticity fixed and varied the storage and release capacities. Both sets compared deterministic, stochastic independent and Markov chain market designs assuming the simulated reality matched the Markov chain.

For Set A experiments, two storage/release configurations were used with different demand elasticities. The first, labelled "least constrained", had maximum storage and release capacity of 1008. Under this configuration if the market design and simulated reality are both deterministic (i.e., perfect foresight) the VWAP values are 1. Under either uncertainty or constrained storage/release the VWAP may differ.

The other configuration, "more constrained" tested kept storage capacity to 252 (three times the average inflow) and release capacity to 168 (twice the average inflow). For these cases, under perfect foresight, the VWAP ranges from 0.968 (for an elasticity of 2) to 11.2 (for an elasticity of -0.125).

Results from these experiments are displayed in Table 1, which compare the changes in VWAP as elasticity changes for both configurations. The three market designs considered are deterministic (Det), stochastic independent (SI) and Markov chain (MC). It can be seen that when demand is relatively inelastic a stochastic market design can have a large effect on the average prices. The effect appears to be more pronounced when the system is constrained. When demand is highly elastic, the advantage appears to disappear completely.

Table 1: VWAP values as elasticity changes under different market designs for two cases

		Least Constrained			More Constrained		
	Market design	Det	SI	MC	Det	SI	MC
Elasticity	-0.125	25.27	4.124	1.903	255.7	37.37	27.03
	-0.25	1.291	1.204	1.155	3.743	3.256	3.073
	-0.5	1.036	1.028	1.025	1.335	1.325	1.314
	-1	1.000	1.000	1.000	1.002	1.002	1.001
	-2	0.995	0.996	0.997	0.954	0.954	0.954
	-4	0.996	0.997	0.998	0.962	0.963	0.963
	-8	0.998	0.998	0.999	0.976	0.977	0.978

One interpretation of the results for the least elastic case is see that the value to the market of moving from a deterministic market design in the more constrained case (VWAP 255.7) to the least constrained case (VWAP 25.27) is similar to the value in moving to a stochastic (Markov chain) market design (VWAP 27.03). In other words, whatever is economically worthwhile spending on increasing the storage capacity could, for the same gains, be spent on implementing a stochastic market design.

Another way to quantify the advantage of a stochastic market design over a deterministic one is to use the difference in annual net welfare. Using constant elasticity curves poses a minor problem in that a continuous constant elasticity curve has no finite integral for inelastic demand – total benefit is infinite. The total benefit calculated will largely depend on the discretisation used. To avoid this difficulty we compare net benefit differences only and use, as a benchmark, the total traded value, under unconstrained perfect foresight, 1008 for this case. Table 2 illustrates the annual net welfare differences for both cases. The differences here appear to be similar except when demands are most inelastic.

Table 2: Annual net welfare difference between deterministic and Markov chain market designs as elasticity changes.

		Least Constrained	More Constrained
Elasticity	-0.125	512	18,690
	-0.25	36.42	152.4
	-0.5	59.99	21.01
	-1	78.36	6.399
	-2	82.60	2.572
	-4	79.43	5.700
	-8	62.77	2.667

For Set B experiments, an elasticity of -0.25 was used over a range of storage and release capacities. This set of experiments only compared cases where the storage capacity was not less than the release capacity for each period.

Table 3 shows VWAP values for deterministic and Markov chain market designs over a range of storage and release capacities. For each capacity pair, the upper value gives the deterministic market design VWAP and the lower (italics) value gives the VWAP for a Markov chain market design.

Table 3: VWAP for deterministic (above) and Markov chain (below) market designs over different SMAX/RMAX pairs.

			RMAX					
			504	**252**	**210**	**168**	**126**	**84**
SMAX	**504**	Det	1.772	1.772	1.772	1.773	1.818	3.781
		MC	*1.559*	*1.559*	*1.559*	*1.560*	*1.591*	*3.115*
	420	Det	–	1.993	1.993	1.995	2.031	3.951
		MC		*1.766*	*1.766*	*1.768*	*1.795*	*3.188*
	294	Det	–	2.993	2.993	2.998	3.038	4.374
		MC		*2.505*	*2.505*	*2.511*	*2.546*	*3.504*
	252	Det	–	3.736	3.736	3.743	3.794	4.732
		MC		*3.065*	*3.065*	*3.073*	*3.123*	*3.869*
	210	Det	–	–	4.929	4.941	5.015	5.647
		MC			*3.984*	*3.997*	*4.074*	*4.692*
	168	Det	–	–	–	6.738	6.875	7.617
		MC				*5.565*	*5.701*	*6.386*
	126	Det	–	–	–	–	9.828	11.081
		MC					*8.685*	*9.789*

As before, an interpretation of the results could be to make observations about similar gains from expanding storage or release capacity compared to those from changing the market design. For example, when RMAX is sufficiently high, the gains from expanding storage capacity from 420 to 504 provide a similar benefit to using a stochastic market design.

Figures 1 and 2 compare the VWAP changes as either SMAX changes for RMAX fixed to 126 (Figure 1) or RMAX changes for SMAX fixed to 504 (Figure 2). Notice that the VWAP is relatively unaffected over a range a RMAX values higher than some threshold. This threshold values is clearly related to the maximum SDQ value (effectively, the largest worthwhile demand). SMAX, on the other hand, has a more gradual effect.

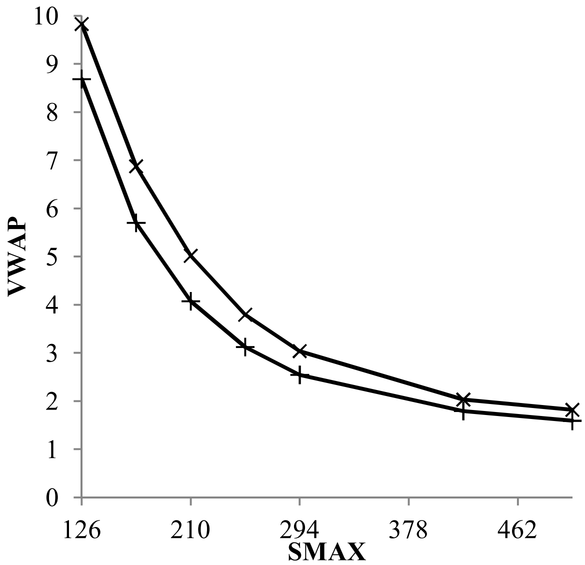

Figure 1: VWAP as SMAX changes with RMAX fixed to 126, for deterministic(×) and Markov chain market(+) designs.

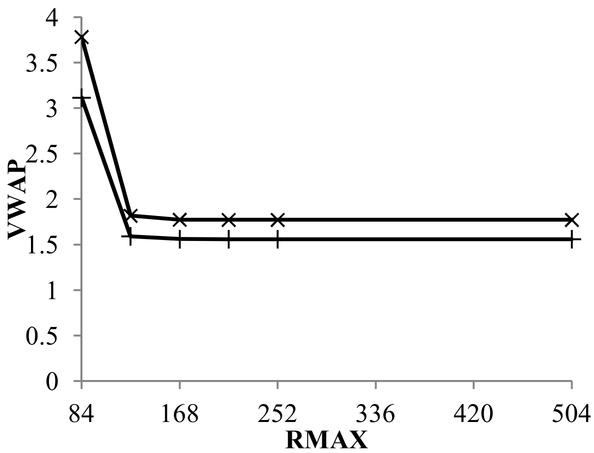

Figure 2: VWAP as RMAX changes with SMAX fixed to 504, for deterministic(×) and Markov chain market(+) designs.

In both cases a Markov chain (+) market design appears to provide an almost constant improvement in VWAP over the deterministic market design (×).

In the following, final sub-set (B) of experiments, we used extreme seasonality. Table 4 compares the improvement in annual net welfare over a range of storage and release capacity values. Again changes in storage capacity appear to have a more significant effect than changes in release capacity.

Table 4: Annual net welfare difference between Markov chain and deterministic market designs over SMAX/RMAX pairs.

		RMAX						
		1008	**504**	**252**	**210**	**168**	**126**	**84**
SMAX	**1008**	35	37	36	35	31	7	18
	504	–	43	32	25	26	15	36
	252	–	–	210	175	160	155	202
	210	–	–	–	426	345	398	360
	168	–	–	–	–	650	646	683
	126	–	–	–	–	–	474	597
	84	–	–	–	–	–	–	440

For specific water system parameter values, annual net welfare cdfs can provide a more detailed comparison. For example, Figure 3 compares cdfs of annual net welfare for the more constrained case with elasticity -0.25. Two market designs are shown, deterministic (broken) and Markov chain (solid). The curves are shifted to remove the large constant benefit, making the lowest additional net welfare value, arbitrarily, zero. This shows that not only does the stochastic market design provide a better expected net welfare, but the advantages are most prominent for low net welfare values – when the available water is lowest and prices highest. The stochastic cdf also appears to show first order stochastic dominance, suggesting that it is preferred regardless of the market's risk attitude. Other cases show similar a comparison.

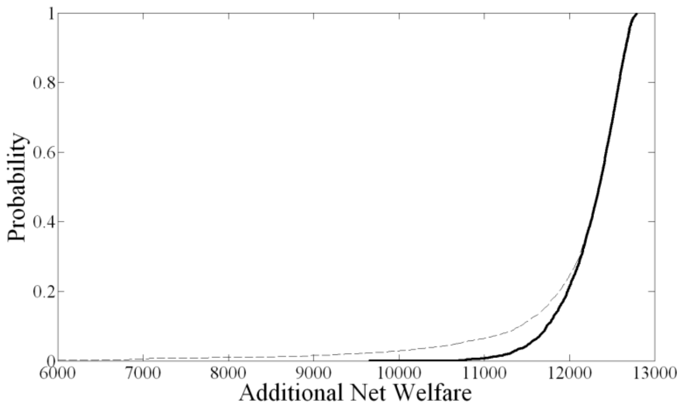

Figure 3: Comparing additional net welfare cdfs from deterministic (broken line) and Markov chain (solid line) market designs.

Care should be taken when comparing the Markov chain results with others. In that case the assumed probability distribution in the market model exactly matches the assumed reality. For such a case the (optimised) expected benefit from the market clearing model exactly matches the expected benefit from performance evaluation. The other cases will perform worse than this when evaluated against the same assumed reality.

2) *Conclusions*

The main purpose of these experiments was to determine whether there were water systems that would potentially benefit from stochastic market designs. Despite the general nature of the experiments, it seems clear that there are indeed such water systems. Further, the parameter values which show improvement using stochastic market designs appear to include those which might be expected in actual systems.

The benefits from explicitly modelling stochasticity in the market-clearing framework ultimately depend on the supply and demand characteristics of the actual system involved. Still these preliminary results suggest that, at least with constant elasticity demand curves, there may be a value in moving to market designs which include stochasticity. Even so, for some configurations, while deterministic models may produce reservoir management strategies that do not look particularly "good", the negative economic impact seems comparatively modest, and might well be offset by the reduction in transaction costs implied by employing from a simpler trading regime, providing greater transparency, and encouraging greater market liquidity. Both observations suggest that much further work is required to better understand which system configurations would best benefit from including a stochastic structure in the market design.

VIII. Future Research

The real benefits from implementing a water market in any particular situation will ultimately depend on a wide variety of social, political, legal, physical and economic factors. At a minimum, though, an acceptable trade-off must be found between the costs of making trading systems and interfaces more complex, and the costs implied by employing an imperfect approach to market-clearing and reservoir management. One might expect accurate modelling of stochasticity to play a critical role in producing outcomes that will be deemed "acceptable", in the sense that market clearing implies a "good" reservoir management policy, without undue loss of potential economic benefits, or exposure to price volatility.

The modelling and evaluation suite described here provides much opportunity to explore the value of stochastic market design in greater detail and over a wider range of water system characteristics. The system developed could be extended in a number of ways as well. For example, to evaluate the economic effects on individual market participants, perhaps spread over a water network. Extension to multiple reservoirs would also provide valuable insights.

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