Matching Users' Rights to Available Groundwater

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Abstract

The amount of available groundwater in a catchment changes quickly, and the amount of water that users can take sustainably depends on where and when it is taken. However, rights to water tend to be fixed, and obtaining rights to water incurs high transaction costs. As a result, water catchments are over-allocated worldwide.

In this paper, I show how a catchment manager could match users' rights to the available water, in near real time, despite uncertain future inflows, while making effective use of all available hydrological data. The solution uses the framework of a smart market. A smart market is a periodic auction cleared with the help of an optimization model. In addition to market clearing, this model allows a convenient means to adjust initial rights, and the auction revenue reflects the available water relative to users' rights. When the auction is revenue neutral, the catchment may be viewed as allocated perfectly. I suggest several ways in which a catchment manager can find this revenue-neutral allocation, assuming the manager has authority to adjust initial rights.

Key words

Auctions, smart markets, groundwater, water allocation, initial rights

Introduction to the smart market for groundwater

Water availability is a contentious issue. How much is available? Who has the rights to it? Despite a great deal of law on the matter, government still has no sure way to allocate water rights. Indeed, many catchments are tragically over-allocated, where users have more water rights than there is water. The problem is less difficult for reservoir water, which is easily measured, and more difficult for groundwater, which is difficult to measure.

A closely related problem to water allocation is protection of the environment. Who should pay for environmental services? Should users pay government for the right to damage the environment, or should government pay users to protect the environment? In many parts of the world, water use is subsidized, but governments also pay users to protect the environment. These perverse behaviours need overhaul in order to manage the environment sustainably. Unfortunately, government management is restricted by the available tools. Too often, the tools are simple policies that can be put into a few words, such as "use it or lost it," "first come, first served," "the market will sort it out," and, of course, "anything goes."

Increasingly, however, water management authorities are using sophisticated hydrological modelling. In some places, this modelling assists pair-wise water trades. Even with this modelling, catchments are often over-allocated, and as a result, the environment is damaged and water is mis-allocated.

In this article, I will assume that the government is operating a smart market for water as described in Raffensperger et al (2009). (A smart market is a periodic auction cleared by an optimization model, such as a linear program.) I will use this framework to show how government can adjust initial rights so that a given catchment can be perfectly allocated. These results follow well-known principles of linear programming, but, surprisingly, have never been applied to water. I will first discuss some of the relevant literature, give an overview of the smart market for groundwater, and explain the problem of revenue sufficiency. In Section 2, the main body of the article, I present a range of methods to adjust initial rights to the available water. These methods include proportional scaling, quantity adjustments, a method for appropriative rights, financial adjustments, and constraint-based adjustments. We shall see that some existing methods for scaling rights result in surprising and counter-intuitive outcomes. We shall see that users might find proportional scaling to be unfair. Some methods result in unclaimed water, or a junior user obtaining more water than a senior user. The key contribution is the simple insight that the smart market gives the policymaker precise control over how groundwater is allocated. The economics and the operations research are new here mainly in their application to this extremely important problem.

I have omitted a case study for brevity, and instead use a running example. Results are given algebraically, and will be immediately applicable wherever the data can be obtained.

Literature review

The work at hand necessarily draws together a wide range of literature, including management of the commons, mechanism design, and hydrogeology. Proper management of the commons clearly requires a central authority (see for example McAfee (1997), and Baliga and Maskin (2003)), but it may take many years for this uncommon wisdom to become common. The large literature on mechanism design is heavily focused on auctions for single items, with analyses especially of market power. Most of this work seems irrelevant to the problem of allocating initial rights for water. Water is divisible, obviously, so it makes little sense to discuss Vickrey or combinatorial auctions. As for market power, groundwater users are likely to have little of it, because a given catchment is likely to have many participants, and research on protecting the commons has shown that market power declines quickly with the number of market participants (McAfee, 1997; Montero, 2008).

Disegni-Eshel (2005) showed how to allocate initial rights to a group of competitive and non-competitive firms, for free, to balance efficiency with market power. Again, more participants resulted in less market power. The small catchment of Marlborough, New Zealand, for example, has about eight hundred well owners. Groundwater users are likely to have little market power because groundwater is spatially dispersed; a well owner has the most control of water locally, nearest the well, and less control of water further away. While market power still needs study in the proposed smart market for groundwater, I will ignore issues of market power in this article, which is about initial rights.

Initial rights can affect efficiency in the presence of large transaction costs (Stavins, 1995b). However, as I shall explain shortly, I will assume zero transaction costs. Nevertheless, despite Coase (1960), and even assuming only price-taking behaviour, the process for setting initial rights does matter (Neuhoff et al., 2006), even when the overall cap is known. For water, and this is the issue at hand, the overall cap is not known in advance, because the catchment manager does not know how much rain the catchment will get. So, even with zero transaction costs, rights should be adjusted in some manner to the available water.

Thoyer (2006) described three existing mechanisms for rights adjustments in over-allocated catchments, via appropriative rights as in California, exchangeable rights as in Australia, and administrative licenses as in France. She touches on many of the same issues as this paper, such as buybacks. Some researchers have used optimization to adjust initial rights, including for water rights. Wang et al (2007) used network linear programming to allocate initial rights for primarily surface water, accounting for return flows, in-stream uses, reservoir storage rights, and pollution. They calculated "fairness" as weighted water shortage, recognizing different seniorities of rights. Environmental flows were viewed as normal demands. They did not envision their system used for a water market, and rights were not adjusted to the available water; rather, users were allocated quantities short of their rights. Lozano, Villa, & Brännlund (2009) proposed to reallocate pollution emission permits with data envelope analysis, trying to avoid decreases in production and increases in pollutants; a similar procedure could be done with water quantity, but because their method requires private data, it is impractical for market operation.

Markets for water are working in Chile (Hearne and Donoso, 2005), (Hadjigeorgalis and Lillywhite, 2004), the U.S. (Michelsen, 2000), and Australia. The Australians, in particular, have worked hard to develop robust systems (Young and McColl, 2007) for water quantity (Bjornlund, 2003; Brennan, 2006; Brooks and Harris, 2005; Brooks and Harris, 2008; Wheeler et al., 2008), and have studied sediment (Tisdell, 2007) using experimental economics, and integrated water modelling (Zaman et al., 2009). The models are not themselves integral to the market-clearing process; they are used to simulate the effects of trades. By contrast, the models in smart markets actually specify the allocations and prices, allowing much richer and more complex multi-lateral trading than can occur with simple zonal-based auctions such as the Watermove market system (Brooks and Harris, 2005; Brooks and Harris, 2008). Those auctions reduce transaction costs, but they best manage surface water stored in reservoirs, not the almost arbitrarily complex hydrogeology that the smart market can handle. Qureshi et al (2009) estimate the gains from water trading in Australia. Based on the Australian experience, Garrick et al (2009) give an excellent review of the enabling conditions for water markets.

The literature on smart markets for water appears to have started with McCabe et al (1991), Dinar et al (1998) and Murphy et al (2000). More recently, similar work has been done by many of the same authors (Murphy et al., 2009; Murphy and Stranlund, 2003). These papers describe a economics experiments with small stylized surface water problems, but they demonstrate some of the general key concepts of smart market operations. Adding the detailed engineering required for real world problems, Raffensperger, Milke & Read (2009) described a smart market in which users buy and sell rights to take groundwater via a shared pool. Raffensperger & Cochrane (2010) and Prabodanie et al (2009) developed similar markets for impervious cover, and point and non-point source discharges, respectively.

In this article, I will depend on the model proposed in Raffensperger, Milke & Read. That work was inspired by the well-established field of hydrological optimization, which applies math programming to solve hydrogeological problems, such as contaminant remediation such as in Ahlfeld and Mulligan (2000). We found that a smart market design based on hydrological optimization elegantly handles the physics, sustainability, and economics. The shared "pool" of water would be managed by a market manager. The market is a spot market, as it enables frequent trading for physical water rights.

The large literature on electricity markets may appear relevant, due to an almost identical mechanism: a periodic auction cleared by a linear program (see, for example, Read et al (1998)). Despite the similarity as smart markets, electricity and water markets will be quite different. The number of market participants is different by orders of magnitude, and groundwater has only a natural supply. The informed reader may immediately think of the financial trades in the electricity markets, and try to make the analogy to water.

Joskow and Tirole (2000) and Gilbert et al (2004) discussed financial transmission rights and market power. Unlike groundwater, initial rights and the available quantities are clear in electricity markets, as generators have initial rights for generation, and the network operator usually owns the initial transmission rights. In any case, the water rights discussed in this article are not financial ownership of the rent on a constraint, nor contracts about price changes, but rights (options) to take physical water. Following establishment of the market for groundwater, participants will want to develop financial trading instruments, such as to hedge against price fluctuations, but the discussion here is only about the physical rights. (This is similar to the financial instruments associated with electricity markets. An underlying spot (immediate) market for electricity operates often, usually every few minutes. To lower risk, participants can also make pair-wise financial transactions outside the market for electricity; these financial transactions may be to buy and sell insurance – hedges – based on spot prices.)

Most relevant to this article, and water allocation, is that a smart market can slash transaction costs, while managing a range of complicated constraints. This advantage has been long noted in the on-line auction and smart market literature (Alvey et al., 1998; Epstein et al., 2002; Hogan, 2005; Pinker et al., 2003; Rassenti et al., 1982), including that for water (Dinar et al., 1998; McCabe et al., 1999; Murphy et al., 2009; Murphy et al., 2000; Murphy et al., 2006). I will mention additional articles in the remainder.

Review of the smart market design for groundwater

To motivate the main part of the article, this section briefly reviews the smart market for groundwater in Raffensperger, Milke & Read (2009).

Assumptions and terminology

A *user* is someone who takes water for a known purpose, e.g., irrigating a vineyard. All users are within one *catchment*, which is a region of land that is relatively isolated hydrologically from other regions. I assume that government has authority to control users' rights to and use of water, and that government has the means and will to enforce market rules. I assume that water is metered.

I assume that water management authorities can allocate two types of rights, which differ mainly in their term.

- *Quota* is a legal option to take water for a short period, e.g., a week or a month. This quota is administrative permission to a particular user to use a certain amount of water for a particular application for a particular period in time, e.g., a week. Water itself is not traded, but rather permission to take water is traded.

- *Consent* is a legal option to take water at a given location, perhaps with a range of restrictions, for a long period, e.g., 10 to 30 years, or permanently. Consent confers quota automatically.

I assume that a *market manager* operates a spot market (i.e., trades are cleared immediately) for groundwater quota within the catchment. The market manager clears the market with the help of a linear program (GWMarket below) which includes explicit constraints for environmental flows. The market manager may require some reasonable knowledge of users' return flows to calculate the effects of the user's abstraction (a hydrologist's term for taking water). Thus, I assume that consent and quota are for a known purpose. I assume that every user's demand function is independent of the user's quota, and that a deterministic market-clearing model is acceptable. The adjustment process will allow the market manager to respond to uncertainty.

The market does not manage bilateral transactions between participants. Rather, trades are to and from a common pool through the market manager. During bidding for a given auction, users give bids via the auction web site, for each period of the planning horizon, e.g., each week through the end of the season. A consent holder may offer to lease the associated quota to the market manager, and a user without consent may bid to rent quota from the market manager. I will use the terms "buy" and "sell," but it should be understood that these transactions are short-term rentals in a spot market. Then the market manager solves GWMarket and informs users of prices and allocations for all remaining weeks of the planning horizon. Users have an initial quota, so after market clearing, sales or purchases are calculated as the difference of the market allocation to the initial quota.

A person who does not hold consent (or even land) could get information to choose a location for operations. To enter the market, a new user could review a map, made available by the market manager, showing prices by location (adjusted for type of use) within the catchment, seeking a well with a relatively low price for water. The user then applies to the market manager to be accepted into the market, satisfying such criteria as reasonable use for the water, sufficient financial stability for the transactions, notification of the location of the user's well, guarantees about metering, etc. The new user does not need to apply for a quantity, but instead can simply rent quota from the market.

The linear program

I next introduce the gross pool market-clearing model for groundwater.

Indices

$b=1,\ldots, B$, demand tranche.

$i, j = 1, \ldots, I$, users, assumed to be in one-to-one correspondence with wells.

$t = 1, \ldots, T$, time period. Period 1 is the present period (e.g., week). Each auction occurs at the beginning of period 1, and the subscript is updated by an external program. Period T is the last period of the season or hydrological year. Thus, the first auction of the hydrological year could have 52 weeks; the second auction could have 51 weeks, and so on, until the final auction is for week $T=52$ only. (A rolling horizon, may be more appropriate (Raffensperger et al., 2009), where at each auction period 1 is re-set to the current date, so T advances into the future.)

u, counter for elapsed time periods for hydrological impacts (constraint set 4).

Parameters

C_i^t = initial quota for user i in time period t. This is not used in Model GWMarket below, but is required to calculate net sales following market clearing. Managing this initial quota is the key issue addressed in this article.

$F^t_{i,k}$ = drawdown rate at control point k, $t - 1$ periods after abstraction at well i, in [meters of head] per [cubic meter of water per second]. Note that $F^1_{i,k}$ is the drawdown in the period of abstraction, not necessarily in period 1. Almost always $F^t_{i,k} \geq 0$. Hydrologists can obtain $F^t_{i,k}$ with standard hydrology software (Ahlfeld et al., 2005).

$P^t_{i,b}$ = the bid price for demand quantity $Q^t_{i,b}$ at user i, tranche b, period t.

$Q^t_{i,b}$ = the bound on demand tranche b for abstraction by user i, period t.

U_k^t = the upper bound on drawdown at control point k, period t, in meters.

V = the minimum net revenue that the market manager requires from users for a given auction. This is not used in Model GWMarket below, but will be used in the market clearing process. V need not be positive if the market manager is willing to make a net payment to users. V is positive, if, for example, the market manager wishes to have net revenue to cover the costs of running the auction. Users will be inclined to ensure that the market manager recovers only what is needed for market operation. However, payment for market operations should be seen as separate from adjusting initial rights to match the available resource.

Variables

$p_i^t \leq 0$, market price per unit of water faced by well i, period t. This is the dual price on constraint 3 below.

$\lambda_k^t \geq 0$, price per unit of head at control point k, period t. This is the dual price on constraint 4 below.

$q^t_{i,b}$ = abstraction at bid price $P^t_{i,b}$ for the well i, tranche b, period t.

q_i^t = total abstraction by well i during period t, a free variable.

Model GWMarket

1. Maximize $\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{b=1}^{B} P_{i,b}^{t} q_{i,b}^{t}$, subject to (1)

2. $0 \le q_{i,b}^{t} \le Q_{i,b}^{t}$, for tranches $b=1,\ldots,B$, users $i=1,\ldots,I$, and periods $t=1,\ldots,T$. (2)

3. $q_i^t = \sum_{b=1}^{B} q_{i,b}^{t}$, for users $i=1,\ldots,I$, and periods $t=1,\ldots,T$. (Dual variable p_i^t.) (3)

4. $\sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1} q_i^u \le U_k^t$, for control points $k=1,\ldots,K$, and periods $t=1,\ldots,T$. (Dual variable λ_k^t.) (4)

Explanation

1. The objective function maximizes users' total welfare from water abstraction over the planning horizon, assuming users bid truthfully.

2. Abstraction in each tranche is bounded by bid quantities.

3. The abstraction equals the total cleared. The dual variable p_i^t of this constraint is the marginal profit to the market for another unit of water taken by well i in period t.

4. The drawdown at each control point is limited, e.g., to prevent coastal salt water intrusion, to maintain minimum stream flows, or to limit aquifer drawdown. This constraint set guarantees that every solution is environmentally sustainable, as defined by the constraints.

I assume $U_k^t \ge 0$, so Model GWMarket is always feasible mathematically. That is, I assume that all environmental constraints can be satisfied when no one abstracts water. The market model manages human-induced impact on drawdown, not natural events.

Because the hydrology data comes from standard hydrology models and software, this system can be set up in any catchment for which a standard hydrology model (e.g. MODFLOW) is available. Many U.S. locations already have such models, and they are often used to help settle legal disputes.

Auction operation

During bidding for a given auction, users specify bids for every remaining period of the planning horizon, e.g., each week through the end of the season. After bids are entered, the market manager solves the linear program, then informs users of the final prices and allocations. Under marginal cost pricing, the user at well i pays $\sum_{t=1}^{T} p_i^t(C_i^t - q_i^t)$, assuming initial rights C_i^t are not scaled or adjusted. Note that the market manager's *required* net revenue V is not automatically the same as the actual net revenue $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(C_i^t - q_i^t)$.

The user would then have firm rights to take up to q_i^1 in the current period, i.e., period 1, and would have quota for q_i^t for periods $t = 2,\ldots,T$. Because all remaining periods are open for bid in every period, users will tend to trade only differences between their adjusted right and what they actually wish to have. More importantly, inflows are uncertain, so future quota are subject to adjustment by the market manager. This problem of adjustment is the main issue addressed in this article.

Revenue sufficiency

Under-allocated catchment

If water is in excess supply, then the constraints in set 4 are slack, so $p_i^t = 0$ and $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(C_i^t - q_i^t) = 0$. In this case, neither users nor policy makers have much interest in market operation. For this article, I therefore assume that water is generally short.

Over-allocated catchment

Worldwide, governments have given more quota than is sustainable, whether because environmental standards have tightened, because catchments face increased drought due to climate change, or because governments have been careless. The case of an over-allocated catchment is important, as it is the root of the world water crisis.

If $\sum_{i=1}^{I}\sum_{t=1}^{T} C_i^t > 0$, feasibility ($q_i^t = 0$ in the most extreme situation) may require that some users are willing to sell. In this case, our design requires the market manager to buy quota from users – paying users to forego water – to protect the environment. Over-allocation may not be a problem in the short run if users are willing to sell for a "reasonable" price. If $V << 0$, the manager pays out large amounts, and the market may be viewed as a procurement (or reverse auction) where users are paid not to damage the environment.

Governments sometimes buy rights from users to protect the environment (McGauran, 2007). Cummings, Holt & Laury (2004) developed a web-based auction system for government to pay farmers to reduce irrigation during drought, in the Flint River Basin, Georgia. The authors' system was not a water trading system, but an irregular drought management system without hydrology modelling. The budget was set in advance, and government accepted the lowest bids until the budget was gone. However, the environment absorbed the difference between the government's decision and any over-allocation. Similarly, Stoneham et al (2003) described the Australian government's environmental services procurement program.

Government has given too many rights to commerce, and now must buy back those rights to protect the environment.

If the catchment is over-allocated, users may realize that they can demand a high price. This is likely to happen as a user observes that her offer to sell was accepted, so she raises the offer price and observes that she improves her revenue. This effect would be enhanced if users collude, but users may observe the effect independently over many auctions, especially if the catchment is over-allocated regularly. The users are likely to catch on even faster if the market manager operates tentative auctions to give users opportunity for price discovery. An over-allocated catchment can result in high payments from the market manager to users, and it is easy to show that under modest assumptions, for a large catchment with many users, these payments can be arbitrarily high.

If users set $P^t_{i,b}$ to a high enough price that $\sum^{I}_{i=1}\sum^{T}_{t=1} p^t_i(C^t_i - q^t_i) < V$, then the market manager cannot clear the market within budget. In this case, I will call the catchment *over-allocated* and will say that the associated auction model is infeasible.

Giving people rights to assets which do not exist is likely to be bad policy. Paying people to forego environmental damage is politically expedient in the short run, but such payments cannot be viable over a long term through a range of economic conditions. Eventually, perhaps due to public outcry, the government will stop paying business not to damage the commons.

If local law allows the market manager to adjust C^t_i, policy makers can choose V as part of auction design and operation. As we shall see, to find a feasible solution at reasonable cost to the market manager, the manager requires authority to adjust quota. At first look, this assumption may appear to be extreme. However, water authorities all over the world routinely tell users how much water they are allocated at the beginning of the agriculture season or year, though mainly with surface water. In addition, water authorities sometimes require well users to shut off on short notice, such as to prevent coastal salt water intrusion. So this assumption is well-grounded in reality.

I define a parameter corresponding to this adjustment.

α^t_i = % of initial quota C^t_i available to the user at well i in period t.

In the following, I will occasionally change subscripts on α^t_i. For example, the scalar α implies the same scaling for all users in all time periods.

If the market manager adjusts quota, then net revenue can be chosen by setting α^t_i in such a way that revenue exactly equals the desired target V, after auction clearing: $\sum^{I}_{i=1}\sum^{T}_{t=1} p^t_i(\alpha^t_i C^t_i - q^t_i) = V$. Clearly, α^t_i

allows flexibility as to which users will get the most adjustment. How should α_i^t be set? Which users should be allocated the most rights? I shall address this shortly.

Optimal allocations and prices are independent of initial allocations

Model GWMarket results in the same flows q_i^t and the same prices p_i^t, for any feasible quota allocation C_i^t, except for alternative optima, and assuming all users participate. Mathematically, this follows trivially because buy and sell quantities are calculated after the model is solved. We also need the assumptions that every user's demand function is independent of the user's quota, and is willing to bid in such a way that $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha_i^t C_i^t - q_i^t) \geq V$, as stated earlier. Economists will note that Coase (1960) proved the same theorem more generally, assuming sufficiently small transaction costs. We should not pass over the assumption too quickly.

Existing water markets worldwide have high transactions costs due to the need to find a trading partner, negotiate and enforce a contract, and manage externalities through protracted government approval processes. The smart market eliminates these transactions costs (users need not see trading partners, negotiating is eliminated, the optimization replaces manual government approvals, and the auction operator manages contracts and trades), thus satisfying the assumptions for Coase's theorem. Hence, if all users participate, the market manager need not re-solve Model GWMarket to find the preferred α_i^t.

To relax the assumption that all participate, we can require that non-participants and participants are scaled similarly. Otherwise, a non-participant could get 100% allocation for free, while the participant gets scaled to 80%, then buys back to 100%, in which case the participant would have been better off without the market. The effect of non-participants is to fix $q_i^t = \alpha_i^t C_i^t$ in Model GWMarket. These users are actually participating, in that they will be taking water; they simply have bids with a sell price of infinity and a buy price of zero. Yet scaling is done *after* the market model is solved, to achieve the revenue goal V. There is a "chicken-and-egg" problem of requiring α_i^t before the model is solved, but calculating it afterwards. This issue may be resolved through a variety of mechanisms, including requiring all users to participate (perhaps for sufficiently large C_i^t and ignoring those below that), using approximate scaling factors for non-participants (perhaps a bit on the low side to incentivise participation), simply charging each non-participant i the amount $p_i^1(1 - \alpha_i^1)C_i^1$, or solving Model GWMarket iteratively to find the right α_i^t.

Methods of adjusting quota

This section describes several methods to choose α_i^t so that the auction model is feasible, i.e., $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha_i^t C_i^t - q_i^t) = V$.

User pays method, revenue positive

Perhaps the most obvious value for α_i^t is $\alpha_i^t = 0$. This corresponds to a user pays market, where $\alpha_i^t C_i^t = 0$ for all i and t. In this method, $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha_i^t C_i^t - q_i^t) = -\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t\, q_i^t = V >> 0$, and the manager receives a large revenue. The market may be viewed as an auction in which users buy all quota from the market manager. (For simplicity of exposition, I will continue to use the word auction to mean one event of this market's operation, regardless of the manager's revenue.) Auctions for public resources are, of course, widely implemented. The revenue raised can be used to offset other distorting taxes, as could be done with carbon emissions (Cramton and Kerr, 2002); money received for environmental services can be used to reduce income taxes, for example.

It is important to remember, however, that water has already been allocated almost everywhere. If $V >> 0$, existing users may not want to implement the proposed market, because they face a significant outlay. If government no longer issued consents for water, prospective users would wish to have the market, as it would give them an opportunity to get quota for water. With centuries of hard-won rights already in place, I think a user-pays system will be unacceptable to users.

Even if we assume a user-pays system, the need to scale rights to the available water remains. Reservoir and groundwater have storage components, so the effects can lag over many periods. Furthermore, users will want some certainty over the full growing season at least, and would not want to commit to a crop without a belief that water will be available for those crops. The allocation problem is therefore multi-period. After period 1, the market manager has granted quota to every user for future periods 2,…,T. Thus, when period 2 arrives, users will already hold quota for which they paid in period 1. However, period 2 flows are unlikely to match their forecast, so the market manager will have to adjust rights again in some fashion, and the scaling problem reappears.

Proportional adjustment

People sometimes like to allocate goods proportionally, out of appeals to "fairness." We can adjust quotas proportionally in a variety of ways.

Myopic maximum proportion

One intuitive method of scaling initial rights is to find the maximum proportion α that all users could take sustainably, found with Model MaxProportion below. I drop the subscripts and subscripts on α_i^t, because all users have the same proportion. Denote the solution to MaxProportion as $\underline{\alpha}$.

5. Model MaxProportion: maximize $\underline{\alpha}$, subject to (5)

6. $q_i^t = \underline{\alpha}C_i^t$, for all wells $i=1,\dots, N$, and periods $t=1,\dots, T$, (6)

and constraints 4.

Because I assumed that a feasible solution exists with $q_i^t = 0$, then MaxProportion has a feasible solution. Furthermore, if the optimal $\underline{\alpha}$ is positive and if all $C_i^t > 0$, then every user is guaranteed some initial quota.

Now the thoughtful reader may think that the lesson from this exercise is that quota should not be a quantity, but rather a fraction of the available water. Another guess is that net revenue to the market manager will be zero, so $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\underline{\alpha}C_i^t - q_i^t) = 0$. Neither guess is true.

Consider the following example, which will appear throughout the remainder of the article.

7. Control point 1: $q_1 + q_2 \leq 10$ (7)

8. Control point 2: $q_2 + q_3 \leq 12$ (8)

The solution (10, 0, 12) maximizes q_1; it also maximizes q_3.The solution (0, 10, 2) maximizes q_2. These solutions allocate different quantities of water, 22 units in the first solution, and only 12 units in the second. Even for a trivial problem, the sustainable available water depends on *where* it is taken. For real problems, available water also depends on *when* it is taken; a price may be high now to prevent future damage. Furthermore, available water changes quickly and variably over space and time. Thus, speaking of the fraction of available water makes little sense. Consent and quota should not be registered or recorded as fractions of some notional “total water in the catchment”.

Regarding net revenue, suppose $C_i = 4$, 8, and 4 for users 1, 2, and 3 respectively. The catchment is over-allocated at control point 1, because $C_1 + C_2 = 4 + 8 > 10$, but is perfectly balanced at control point 2, because $C_2 + C_3 = 8 + 4 = 12$. The solution to Model MaxProportion is $\underline{\alpha} = 10/12$, based on control point 1. User 3 will complain, because she has responsibility only to control point 2, which was perfectly allocated. In the market, she would have to buy back her original quantity, if she wanted it.

If each user bids \$1/unit quota, the solution is (10, 0, 12). User 3 *does* buy back her original allocation, plus more. The dual prices are $p_1 = -\$1$, $p_2 = -\$2$, $p_3 = -\$1$. Net revenue to the market manager is –\$1(4*10/12 – 10) + –\$2(8*10/12 – 0) + –\$1(4*10/12 – 12) = \$2. Thus, the auction is revenue positive.

In general, Model MaxProportion will choose $\underline{\alpha}$ based on the “driest” well. This is most likely the user closest to a particular most-stressed environmental control point. The optimal $\underline{\alpha}$ and the suspect user may be identified as follows.

Substitute $q_i^t = \underline{\alpha}C_i^t$ into constraint set 4: $\sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1}\underline{\alpha}C_i^t = \underline{\alpha}\sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1}C_i^t \leq U_k^t$, and so:

9. $\underline{\alpha} = \min_{k,t}\left\{U_k^t \Big/ \left(\sum_{i=1}^{I}\sum_{u=1}^{t} F_{i,k}^{t-u+1} C_i^t\right)\right\}$. (9)

Denote by (k^-, t^-) as the control point k^- and time period t^- which most constrain $\underline{\alpha}$. The user with the "driest" well is then user $i^- = \operatorname{argmax}_i \sum_{u=1}^{t^-} F_{i,k^-}^{t^- -u+1} C_i^{t^-}$, i.e., the user with the largest impact on control point k^- at time period t^-.

Questions of fairness arise. All users' quota have been reduced to that of the driest user, but if this user i^- took one unit less, users further away from the environmentally sensitive areas could have more than a unit of water. As a result, $\underline{\alpha}$ almost always results in a revenue-positive auction, because most users will buy back some quota.

User trades, well quota proportion, revenue neutrality

A particularly interesting case is where $V \approx 0$. In this case, the market manager is serving as a broker to clear the market, maintaining no financial position; the catchment may be viewed as perfectly allocated. Users are likely to participate voluntarily, unilaterally choosing to buy or sell without government coercion. Non-participating users are likely to have a marginal value for water that is different to the price, and thus they will have incentive to trade. Thus, the user trades method gives the policymaker a simple way to start and expand the market.

The α^0 we seek, indicating revenue neutrality with the superscript 0, solves the following equation.

10. $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha^0 C_i^t - q_i^t) = V = 0$. (10)

This is easily solved: $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t(\alpha^0 C_i^t - q_i^t) = \sum_{i=1}^{I}\sum_{t=1}^{T} (\alpha^0 p_i^t C_i^t - p_i^t q_i^t)$

$= \alpha^0 \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t C_i^t - \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t q_i^t = 0$, which implies

11. $\alpha^0 = (\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t q_i^t) / (\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t C_i^t)$. (11)

Using our earlier example, the solution (10, 0, 12) maximized $q_1+q_2+q_3$, supposing identical bids. Now solve for α^0:

$-\$1(4*\alpha^0 - 10) + -\$2(8*\alpha^0 - 0) + -\$1(4*\alpha^0 - 12) = 0$, so $\alpha^0 = 11/12$.

Users 1, 2, and 3 buy 6.333, –7.333, and 8.667, respectively. Users 1 and 3 buy back only 1/12th of their original quota. User 2 sells everything, but is paid for only 11/12ths. Still, we can imagine user 3 still complaining (as control point 2 was never over-allocated), just not as loud as with the solution to Model

MaxProportion. The same problem of unfairness occurs, but to a lesser degree only because less money goes to the market manager.

Every user will prefer the user trades proportion (where the auction manager's revenue is 0) to the myopic proportion (where the auction manager's revenue is likely to be positive). This is easily proved, and it suffices to show that $\underline{\alpha} \leq \alpha^0$. Simply observe that by linear programming duality, Model GWMarket simultaneously maximizes $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t q_i^t$ and minimizes $\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^t C_i^t$. Therefore, the ratio of these two terms is maximized by the optimal GWMarket solution q_i^{t*}, which means α^0 is the largest possible ratio, and the result follows. Intuitively, because the myopic proportion $\underline{\alpha}$ is based on the tightest constraint in set 4, $\underline{\alpha}$ is likely to be quite small, say, 10%. If the initial quota are remotely close the right value, then the user trades proportion α^0 will be close to 100%. With a larger proportion of initial consent in the user trades method, sellers receive more money, buyers pay less money, and the market manager receives less money.

The user trades method may be viewed as distributing the scarcity rents among the users. In a market where binding constraints correspond to private assets, such as power line capacity in an electricity market, this point of view makes sense. The owner of the capacity has incentive and compensation to increase capacity. (Interestingly, in New Zealand, the electricity network operator Transpower returns a sizeable fraction of line capacity rents to their customers (ECNZ, 2008).) However, environmental constraints may be viewed as public goods; the "capacity" is the allowable draw down of the environmental resource. Government could allocate all sustainable water to users, but (barring technological changes that expand the sustainable capacity of the environment) should not allocate more than that.

User trades, proportion by time period

The market manager could choose to achieve revenue neutrality over a sequence of auctions, say, a year of weekly auctions. Subscripting by time, the manager would wish $\sum_t V^t \approx 0$. However, once the precedent was made for scaling quota, there seems no reason for the market manager to take on the risk. The market should signal information to users about the future levels of quota. To provide this signal, the market manager can choose a method of scaling to have revenue neutrality in every period of the current auction model. This is done easily by adding a subscript for time, α_t^0, then solving for α_t^0.

12. $\alpha_t^0 = \sum_{i=1}^{I} p_i^t q_i^t / \sum_{i=1}^{I} p_i^t C_i^t$, for $t=1, \ldots, T$. (12)

In this way, a given user's quota may be scaled differently for each future period. Thus, in period 1, if a user's quota for period 2 were scaled by $\alpha_2^0 = 15\%$, the user is signalled that period 2 is expected to be relatively dry.

Quantity and financial adjustments

Rather than adjust users' quota proportionally, quota could be adjusted by quantity. Again, this may be done in several ways: minimizing the amount that each user must give up, which would tend to hurt small users, and would not necessarily be revenue neutral; minimizing the amount each user gives up, but in a revenue neutral way (which would still hurt small users); and restricting the user with the highest impact, following the earlier notion of "driest well".

We can also define adjustments based on final net payments, where the market manager achieves revenue neutrality by charging users directly rather than changing their initial quota, like a sales tax. Similarly, the auction manager could charge users a fixed dollar amount. Unless the initial quota is adjusted, users do not receive direct signals about how their quota will be adjusted in the future. So this method seems appropriate to raise revenue to cover the auction costs, not a way to match rights to the available water.

I will omit these easy extensions for brevity. Two remaining quantity methods of interest include maximizing total abstraction and appropriative rights.

Maximize total abstraction

We could begin with the solution that maximizes total water abstraction from the catchment. To maximize total abstraction, we can solve Model MaxWater below, which will give an upper bound on the amount of water available from the catchment. Model MaxWater will be familiar to many hydrologists, as maximizing total flow subject to constraints is a typical hydrological optimization problem.

13. Model MaxWater: maximize $\sum^{n}_{i=1}\sum^{T}_{t=1} q_i^t$, (13)

subject to constraints 4.

The market manager then sets $\alpha_i^t = q_i^t / C_i^t$.

Following our three-user example, this solution would change user 1's initial allocation from 4 to 10, user 2's allocation from 8 to zero, and user 3's allocation from 4 to 12. Users 1 and 3 would be pleased, and user 2 is likely to begin court proceedings. This simplistic method ignores any initial right and willingness to pay. Users closest to environmental control points will be most restricted, losing their quota without regard to previously hard-won battles. Given the decreasing marginal utility for water, this method will tend to hurt some users financially more than others.

Appropriative rights

U.S. laws in some states specify that rights are acquired by use, and the earliest user has senior rights to later users. In dry years, senior users get water first; junior users cannot "hurt" senior users. Users are required to use water “reasonably”, and users argue about what “reasonably” means, especially in dry years, with resolution through the courts. Appropriative rights are a special kind of quantity adjustment.

To create a market for users with appropriative rights, we can adjust Model MaxProportion. Assume that each user i has some C_i^t at which user i is considered to have full quota. Create an α_i for each user, for simplicity dropping the subscript for time. Number the users by seniority, so the senior user is user 1. Beginning with the senior user, for each user j, maximize α_j sequentially:

14. Model Appropriative: for $j = 1, \ldots, I$, maximize α_j, subject to (19)

15. $q_i^t = \alpha_i^* C_i^t$, for all wells $i=1,\ldots,j-1$, and periods $t=1,\ldots, T$. (20)

16. $q_j^t = \alpha_j C_j^t$, for all periods $t=1,\ldots, T$. (21)

17. $q_i^t \leq 0$, for all wells $i=j+1,\ldots, N$, and periods $t=1,\ldots, T$. (22)

18. $0 \leq \alpha_j \leq 1$, (23)

and constraints 4.

Fix α_j to its optimal value α_j^* and go to the next j.

Following this initial allocation, the market manager would run the auction as usual with Model GWMarket. Following that run, the market manager may adjust quota again to achieve revenue neutrality.

This method ensures that the senior user has absolute first use. Another interpretation of appropriative rights instead requires that the most senior user obtains at least as much water as the next junior. This can be done by adding constraints $\alpha_i \geq \alpha_{i+1}$, rather than solving for α_i for each trader. Which method is correct? Similarly, the issue of timing of rights must be addressed: is user 1’s right to period 2 water superior to user 2’s right to period 1 water? This may depend on the length of the period; probably for a day, but perhaps not for a month. Such questions will be answered in the smoke-filled conference rooms and stolid courthouses. In any case, the appropriative doctrine will provide a windfall for senior users. Over time, however, after consent trades hands, the notion of appropriative rights will become meaningless. States have appropriative water rights because government has not worked out how to operate water markets.

Following our example, look again at equations 7 and 8 repeated here.

19. Control point 1: $q_1 + q_2 \leq 10$ (7)

20. Control point 2: $q_2 + q_3 \leq 12$ (8)

Suppose users are numbered in increasing seniority, so user 3 is most senior. Further, assume that each user is deemed to have “full quota” with 7 units. Then user 3 would obtain 7 units, user 2 would receive 5 units (using up the water at control point 2), and user 1 would receive 5 units (using up the remaining water at control point 1). This follows our intuition that the senior user will get the most water.

However, suppose users are numbered in decreasing seniority, so user 1 is most senior. Now user 1 would get 7 units, user 2 would get 3 (using up the water at control point 1), and user 3 would get 7 ("full quota"), leaving 2 units at control point 2 unallocated! Though user 2 was senior to user 3, user 3 got more water. Seniority does not always confer more water, because the market manager must consider water availability. The unallocated water is a bigger surprise. Allocation by seniority is unlikely to result in an optimal solution to Model MaxWater.

Financial adjustments

We can define adjustment methods based on final net payments. The market manager could choose to achieve revenue neutrality by charging users directly rather than changing their initial quota. This may be viewed as a sales tax τ_i^t.

21. $\tau_i^t = p_i^t \,|\, C_i^t - q_i^t \,|\cdot \sum_{j=1}^{I} p_j^t (C_j^t - q_j^t) \;/ \sum_{j=1}^{I} p_j^t \,|\, C_j^t - q_j^t \,|$, for all *i* and *t*. (24)

The sales tax rate is $\sum_{j=1}^{I} p_j^t (C_j^t - q_j^t) \;/ \sum_{j=1}^{I} p_j^t \,|\, C_j^t - q_j^t \,|$.

Following our example, the optimal solution was:

User	C_i	p_i	q_i	User gain
1	4	–\$1	10	–\$6
2	8	–\$2	0	\$16
3	4	–\$1	12	–\$8.

Here, $\sum_{j=1}^{I} p_j^t (C_j^t - q_j^t) = \$2$, and $\sum_{j=1}^{I} p_j^t \,|\, C_j^t - q_j^t \,| = \$30$. The market manager can therefore require additional payments of \$12/30, \$32/30, and \$16/30, from users 1, 2, and 3, respectively, thus achieving revenue neutrality. This method charges all participants, including those with no initial quota. It does not specify a way to scale non-participants.

The extension to fixed charge payments corresponds to a fixed transaction cost on trades: charge each user $\sum_{j=1}^{I} \sum_{t=1}^{T} p_j^t (C_j^t - q_j^t)/I$. Other extensions include the option to charge only buyers, or only sellers, or to adjust each user's initial quota C_i^t to imply the tax τ_i^t, e.g., 4 – 12/30, 8 – 16/30, 4 – 16/30 for each respective user in our example. Indeed, the market manager could use any of many possible methods to achieve the revenue target, but not every method would relate users' rights to the available resource. Unless the initial quota is adjusted, users do not receive direct signals about how their quota will be adjusted in the future. So this method seems appropriate to raise revenue to cover the auction costs, not a way to match rights to the available water. We shall see next a method which relates the users' rights much more closely to the resource than any thus discussed.

Constraint quota

To now, I have described rights in terms of the user, and his or her location: the quantity of water that can be taken from the user's own well. Rights can instead be defined in terms of the user's impact on the control point.

Montgomery (1972), in the context of air pollution, defined "ambient" permits, perhaps an unfortunate name, where the regulator specified separate rights for each control point. A user wishing to discharge at his or her own location had to obtain sufficient separate rights for each control point that his or her discharge would impact. Montgomery's method allowed for multiple control points, in theory.

Systems based on "trading ratios" (Horan et al., 2002; Kerr et al., 2007) have few control points, ideally one, and sometimes using zonal trading ratios, where all users in a given zone have the same ratio. These ratios correspond to oversimplified $F^t_{i,k}$ coefficients in Model GWMarket. In a way, the rights adjustment methods proposed here are similar to these trading ratio systems, in that the right is usually defined in terms of the user's behaviour, not the user's impact. Montgomery's ambient permit system defined rights in terms of the user's impact, not their behaviour.

Of course, even with only one control point, these systems suffer from high transaction costs. Market designers oversimplified the physics in an attempt to reduce the transaction costs, understanding that by doing so, they were creating fewer commodities, which should allow easier trade. But the transaction cost was not nearly reduced enough (Stavins, 1995a).

The smart market suffers from none of these limitations. Linear programming can easily manage a rich set of control points and impact factors, which affect a given control point dynamically over time. The auction format provides a venue for the market. The market clearing process assures the regulator that the commons is respected. And the model formulation lets us choose whether to specify rights based on the user's behaviour or the user's impact. So let us now explore the latter.

Quantity constraint quota

Denote the dual price of constraint (*k*, *t*) of set 4 as λ_k^t. (Recall T as the number of time periods, I as the number of users, B as the number of bid tranches, and K as the number of control points.) From linear programming duality, $\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{b=1}^{B} P^t_{i,b} q^t_{i,b} = \sum_{t=1}^{T}\sum_{k=1}^{K} \lambda^t_k U^t_k$. Thus, the value of the right to take water corresponds to the value of the resource. This insight changes the notion of a right to take water into a right to reduce head at a set of control points. This latter interpretation corresponds to a claim of $F^{t}_{i,k}C^{t}_{i}$ against the right hand side $U_k^{\,t}$.

I will use the term *constraint consent* to mean a long-term right to reduce head at a control point, and *constraint quota* $C^{t}_{i,k}$ to mean the quantity of right of user *i* to reduce head at control point *k* in period *t*. A constraint quota is for a single period and a single control point. Abstraction of water from a given well will likely affect many control points over many time periods. To take a unit of water, the user at well *i* must have quota to every control point where $F^{t}_{i,k} > 0$. Rather than holding *T* quota, one for each period, the user would be required to hold *KT* separate constraint quota, a right for every control point in every period. The market manager now has:

$$22.\ \text{net revenue} = -\sum_{k=1}^{K}\sum_{t=1}^{T}\lambda_k^t\left(\sum_{i=1}^{I}C^{t}_{i,k} - \sum_{i=1}^{I}\sum_{u=1}^{t}F^{t-u+1}_{i,k}q_i^u\right). \qquad (25)$$

Scaling quota, then, is necessarily different. To achieve revenue neutrality, the market manager may wish to scale users' constraint quota $C^{t}_{i,k}$ by $\alpha^{t}_{i,k}$, in its most general form. It seems appropriate to scale the quota equally for all users at a given control point and time period, so I drop the subscript *i* and use α_k^t. For revenue neutrality,

$$23.\ \lambda_k^t\left(\sum_{i=1}^{I}\alpha_k^t C^{t}_{i,k} - \sum_{i=1}^{I}\sum_{u=1}^{t}F^{t-u+1}_{i,k}q_i^u\right) = 0, \text{ for each control point } k \text{ and time period } t. \qquad (26)$$

The λ_k^t drops out. The scale factor α_k^t that we seek is easily calculated as

$$24.\ \alpha_k^t = \sum_{i=1}^{I}\sum_{u=1}^{t}F^{t-u+1}_{i,k}q_i^u \Big/ \sum_{i=1}^{I}C^{t}_{i,k}\text{ , for each } k \text{ and } t. \qquad (27)$$

Interestingly, α_k^t does not depend directly on the price λ_k^t, except indirectly through the price impact on the solution q_i^t. This is different to the well quota method, which depended directly on the price p_i^t.

Following market clearing, each user *i* would be given firm rights to induce drawdown of $F^{1}_{i,k}q_i^1$ at every control point *k* in the current period, and constraint quota of $C^{t}_{i,k} = \sum_{u=1}^{t}F^{t-u+1}_{i,k}q_i^t$ for each control point *k* and each period t=2,…,T. A non-participating user *i* would be restricted to the tightest constraint: $q_i^1 = \min_{k,t}\{\alpha_k^t C^{t}_{i,k}/F^{1}_{i,k}\}$ for each time period *t*, which would incentivise participation.

Consider again our continuing example. Recall equations 7 and 8 as

Control point 1: $q_1 + q_2 \leq 10$
Control point 2: $q_2 + q_3 \leq 12$

Initial quota were 4, 8, and 4, respectively. Suppose instead that the rights were recorded by control point. Thus, user 1 has rights of 4 to control point 1, and no rights to control point 2. User 2 has rights of 8 to control point 1 and 8 to control point 2. User 3 has rights of 4 to control point 2 only.

Maximizing $q_1 + q_2 + q_3$ as before, the primal solution is (10, 0, 12) as before, and $\lambda_1 = \lambda_2 = \$1$, but those prices will not be required until we calculate the final user payments. We seek a scale factor for each constraint:

For constraint 1, $\alpha_1 = (1*10 + 1*0 + 0*12)/(1*4 + 1*8 + 0*12) = 5/6$.

For constraint 2, $\alpha_2 = (0*10 + 1*0 + 1*12)/(0*4 + 1*8 + 1*4) = 1$.

Because only control point 1 was over-allocated, we need not adjust the rights for user 3. We set $\alpha_1 = 5/6$ and $\alpha_2 = 1$. Table 1 compares the earlier user trades method to this constraint quota method.

Compared to the well quota method, user 1 is slightly worse off and user 3 is slightly better off. In the well quota method, user 3 was restricted due to the over-allocated control point 1, and then bought back the quota; that money offset user 1's restriction. So user 1 obtained a benefit (better than a free ride) at user 3's expense. People are likely to view the constraint quota as fairer than the well quota method, as it assigns the full cost of the environmental impact to the correct users. If environmental standards were changed, then U_k^t changes, and the constraint quota method would specify which users would be affected.

The constraint quota method could be used with a quantity reduction, reducing first those users with the largest impact, $\max_i C^t_{i,k}$ for each k and t. In our example, user 2's constraint quota for control point 1 would be reduced from 8 to 6. The solution (10, 0, 12) is unchanged, but user 1 pays \$6, user 2 gains \$14, and user \$3 pays \$8. Thus, in comparison to the proportional method, user 1 gains at the expense of user 2.

Because each user requires constraint quota, and the solution q_i^t applies, for every relevant control point and time period, we might expect that the constraint quota system would converge over some long run to be identical to the well quota system. This would be true if U_k^t (which reflect natural flows) move together, and in the same fashion that rights are adjusted, which is unlikely. Further, if environmental standards change, the market manager will wish to know exactly which users are affected, and by how much, in order to determine any compensation or rights reductions.

Percent constraint quota

Earlier, it was argued that recording quota as a percent of available water made little sense, because available water depends on where and when it is taken. However, if we specify quota in a way that *does* depend on where and when it is taken, then a percentage quota can work. In this method, we view the constraint quota $C^t_{i,k}$ as a percent of the available drawdown U^t_k, easily calculated from the initial well quota C^t_i.

25. $$C^t_{i,k} = \sum\nolimits_{u=1}^{t} F^{t-u+1}_{i,k} C^t_i \Big/ \sum\nolimits_{j=1}^{I} \sum\nolimits_{u=1}^{t} F^{t-u+1}_{j,k} C^t_j \,. \qquad (32)$$

Note that $\sum_i C^t_{i,k} = 1$. We could have defined $C^t_{i,k}$ as $C^t_{i,k} = \sum\nolimits_{u=1}^{t} F^{t-u+1}_{i,k} C^t_i \big/ U^t_k$, but then this would change when U^t_k changed, and we would also have to rescale $C^t_{i,k}$ so that $\sum_i C^t_{i,k} = 1$.

Following market clearing, each user is given firm rights for the current period to draw down each control point k by the fraction $C^1_{i,k} = F^1_{i,k} q^t_i \big/ \sum\nolimits_{j=1}^{I} F^1_{j,k} q^t_j$, and each user is given quota for periods 2,…,T to draw down control point k by the fraction $C^t_{i,k} = \sum\nolimits_{u=1}^{t} F^{t-u+1}_{i,k} q^t_i \big/ \sum\nolimits_{j=1}^{I} \sum\nolimits_{u=1}^{t} F^{t-u+1}_{j,k} q^t_j$.

Do we need to rewrite constraint 4? No, because all the rights calculations are done after the model is solved. The market manager has

26. $$\text{net revenue} = -\sum_{k=1}^{K} \sum_{t=1}^{T} \lambda^t_k U^t_k \left(\sum_{i=1}^{I} C^t_{i,k} - \sum_{i=1}^{I} \sum_{u=1}^{t} F^{t-u+1}_{i,k} q^u_i \Big/ \sum_{i=1}^{I} \sum_{u=1}^{t} F^{t-u+1}_{i,k} q^u_i \right). \qquad (33)$$

The terms in parentheses cancel as (1 – 1), so the net revenue is guaranteed to be zero. Thus, this method needs no scaling to achieve revenue neutrality. If the market manager wants $V \neq 0$, a range of methods is readily available. This percentage approach simplifies the adjustment process considerably, and would be handy if U^t_k often changes contrary to forecast.

How might users be compensated for tighter environmental standards? In a user pays system, the policy maker could wait for the beginning of the hydrological year to adjust U^t_k. For systems with long-term consents, the policy maker could wait for the consent renewal to adjust U^t_k fractionally, as users' renewals are probably not simultaneous. For systems with permanent consents, the market manager may be tempted to tell users that their rights were defined as a percentage, and they still have the same fraction, just of a smaller pie, and therefore no compensation will be forthcoming. However, once society has made a precise agreement with commerce regarding the rights to the commons, changes to the agreement must follow due process. The policy maker and users, and presumably the courts, would need to give careful attention to the wording of the rights documents.

A revenue-positive market will be disliked by users. With a revenue-neutral market, users would be incentivised to participate voluntarily. A strongly revenue-negative market is almost certainly unsustainable from the market manager's business point of view. With appropriative rights, a given user may get less water than a more junior user, due to local conditions and availability, and sometimes water may be left unallocated. Recording well quota as a percent of "total water in the catchment" is not a good idea, as the water available depends on where and when it is taken. Users will probably view constraint quota to be the fairest method, as free riding is eliminated. In addition, constraint quota could be recorded as a percentage.

Conclusion

Much of the world water crisis can traced to the inability to assign initial rights sustainably. As a result, catchments are over-allocated almost everywhere. This article has demonstrated several important methods to allocate initial rights within the limits of sustainability, using the framework of a smart market. The problem of revenue neutrality is identical to that of matching quota to the available resource. Rather than under or over-allocation, water in the catchment can be allocated perfectly. If rights can be adjusted, these allocation methods will work despite future uncertainty, with complex hydrology. Because the methods proposed in this article depend mainly on linear programming duality, results are not restricted to groundwater, and should apply to many smart markets cleared by linear programs.

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