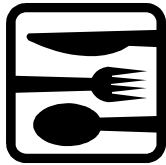




Q-Imports Student Activity



The firm Q-Imports sells 1000 sets of stainless steel cutlery every year. This cutlery is imported from Germany at a cost of \$60 per set, including airfreight and customs duty. Sales occur at a fairly steady, even rate throughout the year. The manager plans to import these in two shipments of 500 sets. For each shipment, the firm incurs clerical costs, bank charges, and customs agent clearing charges, totalling \$200. Q-Imports also incurs costs for storing the cutlery, which amount to \$12 per set per year. Q-Imports want to determine the total annual cost and see whether there is any possibility to reduce this.

We will analyse this situation as an inventory problem using a model often called the EOQ model. Inventory problems arise in most industries and many businesses.

Before we begin to analyse the Q-Imports situation we make two assumptions:

1. Sales occur at a constant continuous rate, so that the number of sets of cutlery in stock reduces in a straight line between imports.
2. The same amount will be ordered with each import.

To develop a formula for the cost (our model) we need to summarise the information we have and define some variables to use in our formulas. Call the yearly demand D , the cost per set v , the imported quantity Q , the shipment charge s and the storage cost c .

1. From the information, we see that $D = \underline{1000}$, $v = \underline{\$60}$, $Q = \underline{500}$, $s = \underline{\$200}$, and $c = \underline{\$12}$.

The total annual cost has three parts: the cost of buying the cutlery sets, the cost of importing the cutlery sets and the cost of storing the cutlery sets. We consider each in turn.

First, the cost of buying the cutlery sets is investigated.

2. What is the cost of buying 1000 sets of cutlery at \$60 per set?

$$\underline{1000 \times 60 = \$60,000}$$

3. What is the cost of buying D sets of cutlery at v dollars per set?

$$\underline{Dv}$$

Next, the cost of importing the cutlery sets is investigated.

4. If we import 500 sets of cutlery 2 times in a year, how many sets of cutlery are imported each year?

$$\underline{500 \times 2 = 1000}$$

5. If we import Q sets of cutlery n times in a year, how many sets of cutlery are imported each year?

$$\underline{Qn}$$

6. We want the total sets of cutlery imported in a year to be equal to the sets of cutlery sold in a year, D . Write an equation that expresses this.

$$D = Qn$$

7. Given $D = Qn$, rearrange the equation to make n the subject.

$$n = D/Q$$

8. If D is the number of sets of cutlery required and Q is the number of sets of cutlery in each shipment, how many shipments are required to ensure that exactly D sets of cutlery are imported?

$$D/Q$$

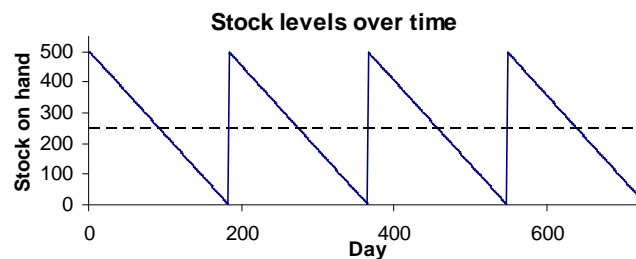
9. If 2 shipments are required and the cost per shipment is \$200, what is the total cost of shipping?

$$2 \times 200 = 400$$

10. If D is the number of sets of cutlery required, Q is the number of sets of cutlery in each shipment, and s is the cost of each shipment, what is the total cost of shipping?

$$sD/Q$$

Finally, the cost of storing the cutlery sets. For this we will assume that the cutlery sets are only imported when Q-Imports have run out of sets in stock. If they order 500 each time, the graph of how many sets they have in inventory will look like this:



The steep increases (from 0 to 500) are the days new cutlery is imported while the more gradual slopes show how the continuous demand gradually reduces the stockpile. The average stock level (shown as the dashed line) is 250 cutlery sets. For a more general import order quantity of Q , the average stock level is $\frac{1}{2}Q$. The storage cost can be calculated on this average stock level.

11. If the average stock level is 250 sets throughout the year and the storage cost is \$12 per set per year, what is the total yearly storage cost?

$$250 \times 12 = 3000$$

12. If the import quantity is Q cutlery sets and the storage cost is c dollars per set per year, what is the total yearly storage cost?

$$\frac{1}{2}Qc$$

[Note for teachers: It is also possible to determine this holding cost using integration over the period between two import days.]

13. The total annual cost to Q-Imports, T , is just the cost of buying the cutlery sets plus the cost of importing the cutlery sets plus the cost of storing the cutlery sets. Write the formula for T using the formulas from questions 3, 0 and 0.

$$T = Dv + sD/Q + \frac{1}{2}Qc$$

14. What is the total annual cost to Q-Imports using the values you determined in question 1?

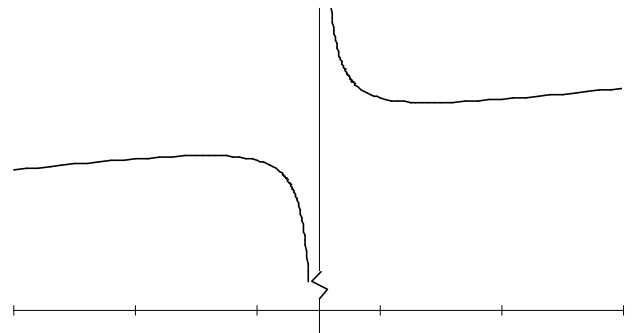
$$T = Dv + sD/Q + \frac{1}{2}Qc = 1000 \times 60 + 200 \times 1000/500 + \frac{1}{2} \times 500 \times 12 = 63,400$$

The problem has two types of variables: *parameters* which the manager has little or no control over and *decision variables* which the manager is free to choose. For this problem the parameters are the yearly demand, D , the cost per set, v , the shipment charge, s , and the storage cost, c . The decision variable is the imported quantity, Q . We can examine how changes in the decision variable change the total annual cost for Q-Imports.

Since we want to treat the total annual cost as a function of the ordered quantity we write $T(Q) = Dv + sD/Q + \frac{1}{2}Qc$ or, using all of the parameter values for Q-Imports, $T(Q) = 60000 + 200000/Q + 6Q$.

15. Enter the value of T from question 14 in the appropriate place in the table below. Then complete the rest of the table for the given values of Q . using the total annual cost formula for Q-Imports, above.

Q (sets)	$T(Q)$ (\$)
100	62,600.00
200	62,200.00
300	62,466.67
400	62,900.00
500	63,400.00



The graph of $T(Q) = 60000 + 200000/Q + 6Q$ appears at the right.

16. Using the equation for $T(Q)$, what happens when $Q = 0$?

The function value is undefined.

17. Where is this represented in the graph?

There is a vertical asymptote at $Q = 0$.

18. For the function $T(Q)$, what is the domain of Q ?

All real numbers except $Q = 0$.

19. Thinking about the definition of Q , what does a value of $Q = 0$ mean?

It means that we never order any sets of cutlery.

20. Why are values of $Q \leq 0$ not appropriate for this situation?

It is not possible to order a negative number of sets of cutlery!

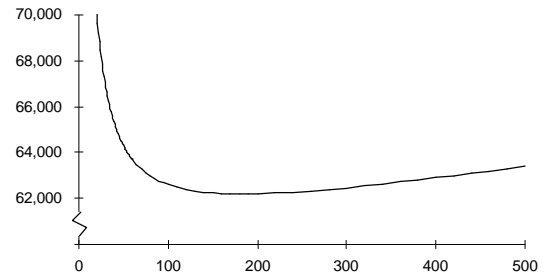
21. Trace the portion of the graph that is appropriate for Q-Imports situation.

Students should trace the part for $Q > 0$

22. Write the domain of the portion of the graph you traced.

$Q > 0$

The graph shows $T(Q) = 60000 + 200000/Q + 6Q$ over a suitable domain. From the graph we see that the minimum value of $T(Q)$ occurs when its slope (derivative) is zero.



23. What is the derivative of $T(Q)$?

$$T'(Q) = \underline{-200000/Q^2 + 6}$$

24. Solve the equation $T'(Q) = 0$ for Q .

$$\underline{T'(Q) = -200000/Q^2 + 6 = 0 \quad \text{P } 200000/Q^2 = 6}$$

$$\underline{\text{P } Q^2 = 200000/6 = 33333.333}$$

$$\underline{\text{P } Q = \sqrt{33333.333} = 182.574}$$

25. Using the result from question 24 for Q what is the total annual cost for Q-Imports of ordering this amount for each import?

$$\underline{T = 1000 \times 60 + 200 \times 1000 / 182.574 + \frac{1}{2} \times 182.574 \times 12 = 62,190.89}$$

26. For this importing quantity, Q , determined above, on average, how many times per year would Q-Imports import cutlery?

$$\underline{n = D/Q = 1000/182.574 = 5.48}$$

27. Compare the total annual cost from question 0 to the total annual cost from ordering 500 cutlery sets with each import (question 14). Discuss whether Q-Imports should change the quantity they order.

Yes, because they can save \$1209.11 per year.

28. Using the more general formula $T = Dv + sD/Q + \frac{1}{2}Qc$, what is the partial derivative of T with respect to Q ?

$$\underline{\frac{\partial T}{\partial Q} = -sD/Q^2 + \frac{1}{2}c}$$

29. Solve the equation $\frac{\partial T}{\partial Q} = 0$ for Q .

$$\underline{T'(Q) = -sD/Q^2 + \frac{1}{2}c = 0 \quad \text{P } sD/Q^2 = \frac{1}{2}c}$$

$$\underline{\text{P } Q^2 = 2sD/c}$$

$$\underline{\text{P } Q = \sqrt{\frac{2sD}{c}}}$$

This formula is known as the “Economic Order Quantity” or EOQ. It is used by many businesses to help to determine the best level of stock to hold.